

Long-Term Slope Stability Modelling and Risk Assessment of Lake Most – Preliminary Results

Modelování dlouhodobé stability svahu a hodnocení rizik jezera Most – předběžné výsledky / Modellierung der langfristigen Böschungsstabilität und Bewertung der Risiken des Sees Most – vorläufige Ergebnisse

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ABSTRAKT

The Lake Most is located in the central part of the Most Basin (Czech Rep.), which is a geomorphological part of Eger graben. The lake was created by flooding of Most-Ležáky residual pit between October 2008 and September 2014. Like in rest of Europe, the safety and the security of mine lakes is one of the priorities of flooding and post-flooding period. One of the main hazards identified is the slope stability of lake banks. To develop a reliability methodology for assessing the long-term stability of flooded open pit mines, a large-scale numerical model of the lake was carried out and was applied on Lake Most. The large-scale numerical model was built, based on the site observations, large scale LiDAR data and geotechnical data. The results highlighted the reliability of the methodology to combine the geometric model with the geological model to create a large-scale numerical model, and to identify local and potentially instable zones.

Jezero Most se nachází v centrální části Mostecké pánve (Česká republika) a je geomorfologickým celkem, který je součástí Oherského riftu. Jezero vzniklo řízeným zatopením zbytkové jámy lomu Most-Ležáky v období říjen 2008 až září 2014. Stejně jako ve zbytku Evropy, je i zde bezpečnost lokality jednou z priorit jak v průběhu, tak po zatopení. Jedním z hlavních identifikovaných rizik jsou svahové deformace postihující svahy a břehy jezera. Pro vypracování metodiky spolehlivosti hodnocení dlouhodobé stability byl zpracován rozsáhlý numerický model, který byl kalibrován na jezeře Most. Na základě monitoringu lokality, rozsáhlých dat z LiDARového průzkumu a geotechnických dat byl sestaven rozsáhlý numerický 3D model celé lokality. Výsledky zdůrazňují spolehlivost metodologie využívající kombinaci geometrického modelu s geologickým modelem pro vytvoření rozsáhlého numerického 3D modelu, určeného pro identifikaci potenciálně nestabilních zón.

Der See Most befindet sich im Zentralteil des Moster Beckens (Tschechische Republik) und ist eine geomorphologische Ganzheit, die ein Bestandteil des Eger Riftes bildet. Der See entstand infolge der geregelten Beflutung des Tagebaurestloches Most-Ležáky im Zeitraum Oktober 2008 bis September 2014. Gleich wie im Rest Europas, gehört auch hier der Standortssicherheit eine der Prioritäten sowohl im Verlauf als auch nach der Beflutung. Ein der grundlegenden identifizierten Risiken sind die Böschungsdeformationen betreffs Seeböschungen und Seeufer. Für die Erarbeitung der Zuverlässigkeitsmethodik bei der Bewertung der langfristigen Stabilität, wurde ein umfangreiches numerisches Modell ausgearbeitet, das an dem See Most kalibriert wurde. Auf Basis Monitoring des Standortes, der umfangreichen Daten aus der LiDAR-Untersuchung und aus den geotechnischen Daten wurde ein umfangreiches numerisches 3D Modell des ganzen Standortes zusammengesetzt. Die Ergebnisse hervorheben die Zuverlässigkeit der Methodologie, die eine Kombination eines geometrischen Modells mit einem geologischen Modell für Bildung eines umfangreichen numerischen 3D Modells nutzt, das für die Identifikation potenzial unstabilen Zonen bestimmt ist.

Klíčová slova: zatopený lom - stabilita svahu - posuzování rizik - LiDAR - numerické modelování
Keywords: post-mining lake - slope stability - risk assessment - LiDAR - numerical modeling

1 | INTRODUCTION AND OBJECTIVE

Almost all post exploitation open-pit mines in Europe and in the world are shaped as a final reservoir intended to be filled with water. These artificial lakes are currently (and in the future) dedicated to recreational purposes, including energy installation purposes (SUMAD RFCS project). To ensure safe utilization of these localities by the public, it is necessary to assess their potential risk of instability. Slope instability is the major long-term hazard. The RFCS RAFF project aims to research issues related to pit lakes. The stability of pit lake slopes after flooding remains an area of uncertainty. Examples of geotechnical failures in slopes and banks of pit lakes are well documented, for example, those at pit lake

Patnów ([1]), Zülpich Mitte and Concordia Lake near Nachterstedt ([2]). Back analysis is generally carried out to understand the cause of these phenomena and to suggest mitigation strategies.

The Lake Most site (Czech Republic) was selected, as a case study, to perform a large-scale stability analyses based on in-situ observation, a 3D geometric model and a large-scale numerical model using strength reduction method. Lake Most (below the hill Hněvín) was formed on the site of the former town of Most, which had to give way to brown coal mining in the second half of the 20th century. Coal mining was ended in August 1999. The lake was established in the open pit mine and the dumps. The flooding up began in October 2008 and

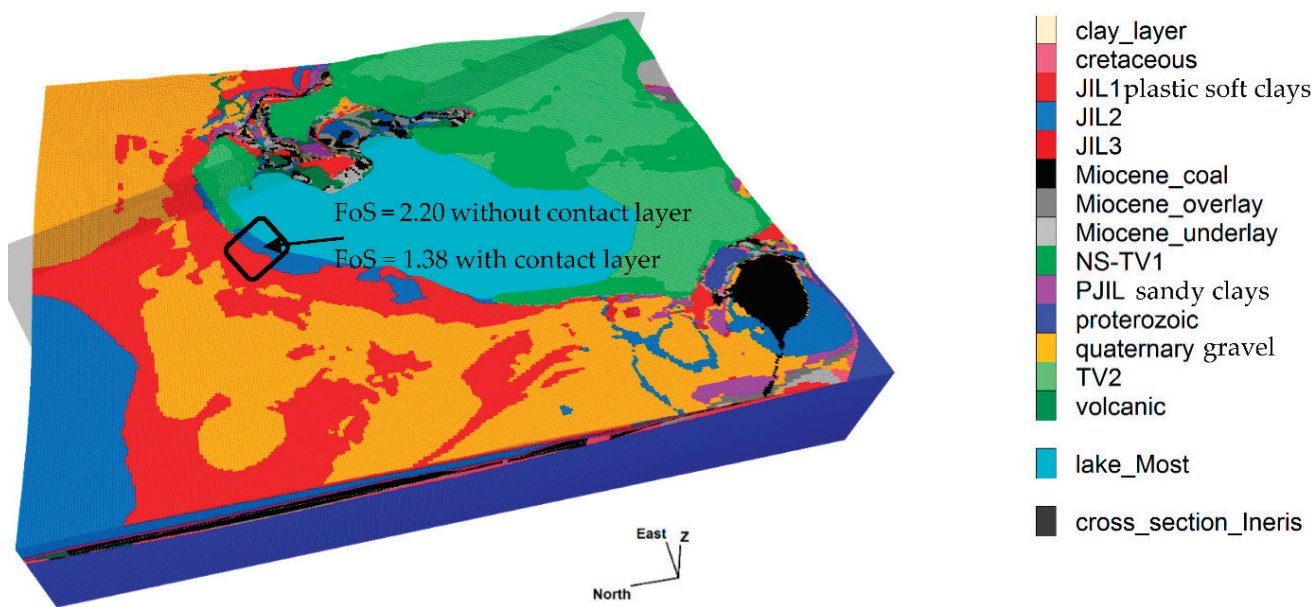


Figure 1: 3D geological model of lake Most (11,826,069 elements); rectangular limit: area where the FoS is minimal.

was finished in September 2014 (surface = 309 ha, volume = 70 hm³; [3]).

2 | LONG-TERM SLOPE STABILITY

To develop a reliability method for assessing the long-term stability of flooded open pit mines, a slope stability study of the lake area was carried out with a large-scale 3D can model complex soil and rock behaviours. The 3D model is based on the site observations, large LiDAR data, geological, hydraulic and geotechnical data.

2.1 | GEOMETRIC AND GEOLOGIC DATA

A 3D numerical model is time-consuming, however, the study area must, at least, integrate the areas of ground movement already identified and

must consider the height of geometric anomalies (valleys or hills). The maximum depth of the lake is 71,1 m, the highest hill in the immediate vicinity of the lake has a height of 70 m and the boundaries of the 3D model was positioned at over 6 times 70 + 75 m from the shores of Lake Most. Additionally, a large LiDAR campaign was carried out in August 2019. The final data point cloud was used to create the digital terrain model to build the 3D volumetric mesh. To avoid too many meshes and therefore prohibitive calculation, a finer mesh corresponding to 5 m was adopted based on 2D calculation for the area of interest whose horizontal perimeter includes all records and observed ground movements in the Most Lake slopes. The elements at the boundaries are cubes of 50 m edge. The 3D geometric model thus, designed has 11,826,069 elements. In total, 14 non tabular geologic formations are identified, and faults are present. The geology of the site is based on thousands of boreholes (carried out between 1867 and 2018) that have been used to build DMT of the interfaces of the different formations. Since the spatial distribution of these boreholes is not homogeneous and does not always extend to the limits of the model, it was necessary to interpolate and extrapolate the position of these interfaces. The analysis of the numerous data of this campaign has helped to determine both the position of these interfaces as well as the values of the geomechanical parameters. Figure 1 represents the different geologic units used for building the 3D numerical model.

2.2 | GEOTECHNICAL DATA

The geotechnical data are based on 2 groups ([3]): the data relating to geological formations and the data characterizing the dump units. The first group concerns all the units described in Figure 3 except NS-TV1



Figure 2: Strímice dump forms the entire eastern slope of lake Most.

$$u_2 + \gamma B \tan \phi + (\sigma_v - u)N_q + \gamma B N_q \tan^2 \phi + C \frac{N_q - 1}{\tan \phi} = q_t \quad (1)$$

$$C = f_s - 0.000789 (1 - \sin \phi)(\sigma_v - u) \tan \frac{2\phi}{3} \left(\frac{q_t - \frac{\sigma_v}{3} (1 + 2(1 - \sin \phi))}{\frac{\sigma_v - u}{3} (1 + 2(1 - \sin \phi))} \right)^{1.44} \quad (2)$$

$$\text{with the bearing capacity factor } N_q = \frac{(\sin \phi + 1)^2}{\cos \phi} \exp \left[\left(0.0061 \frac{q_t}{P_a} + 2.4 \right) \tan \phi \right] \quad (3)$$

$$pd f = \frac{\sqrt{\frac{x}{\beta}} + \sqrt{\frac{\beta}{x}}}{2\sqrt{2\pi\gamma x}} e^{-\frac{x + \frac{\beta}{x} - 2}{2\gamma^2}} \text{ with } \beta = \frac{\mu}{3}(4 - \sqrt{3c^2 + 1}); \gamma = \sqrt{\frac{2(c^2 - 1 + \sqrt{3c^2 + 1})}{5 - c^2}} \text{ and } c = COV = \frac{\sigma}{\mu} < \sqrt{5} \quad (4)$$

and TV2 anthropogenic units. Among these geological units, the 3 least resistant units (quaternary gravel, JIL1 = plastic soft clays and PJIL = sandy clays: $52 \text{ kPa} < \sigma_c < 143 \text{ kPa}$) will play a role in the calculation of slope stability, especially in areas where these units are near the topographic surface. The second group of data is very rich in geotechnical information due to a recent measurement campaign (CPT). There are 9538 CPT measures available, including pore pressure (u_2), sleeve friction (f_s), depth, cone resistance (q_c), corrected cone resistance (q_t), soil weight (γ), density (ρ), pore water pressure (u), total stress (σ_v) and Young's modulus (E). The last 6 parameters are classically calculated from the data recorded every 10 cm and parameters characterizing the used cone (net area ratio: $a = 0.8$ and penetrometer diameter $B = 0.0357 \text{ m}$). As materials are assumed to have a Mohr-Coulomb elastoplastic constitutive model, the cohesion (C) and the friction angle (ϕ) was calculated from these data.

The CPT data interpretation theory manual gives 3 empirical relationships ([4–6]) for assessing ϕ . The disadvantage is that these relationships do not give access to cohesion and that their validity is limited to sands or to fine-grained soils. Therefore, we preferred to use the system of 2 equations proposed by Motaghedi & Eslami ([7]) which gives access to C and ϕ after numerical resolution.

The parameters of the CPT campaign were recorded for 23 exploitable penetrometric profiles whose depth of investigation varies between 10 m and 101 m. This depth was calibrated to the estimated thickness of the mining deposits. These 23 profiles have been statistically analysed with an automated method based on the evolution of the coefficient of determination (between depth and cohesion). The position of the interface between different dump units is got by analysing the cohesion profiles (Figure 3). This completes the determination

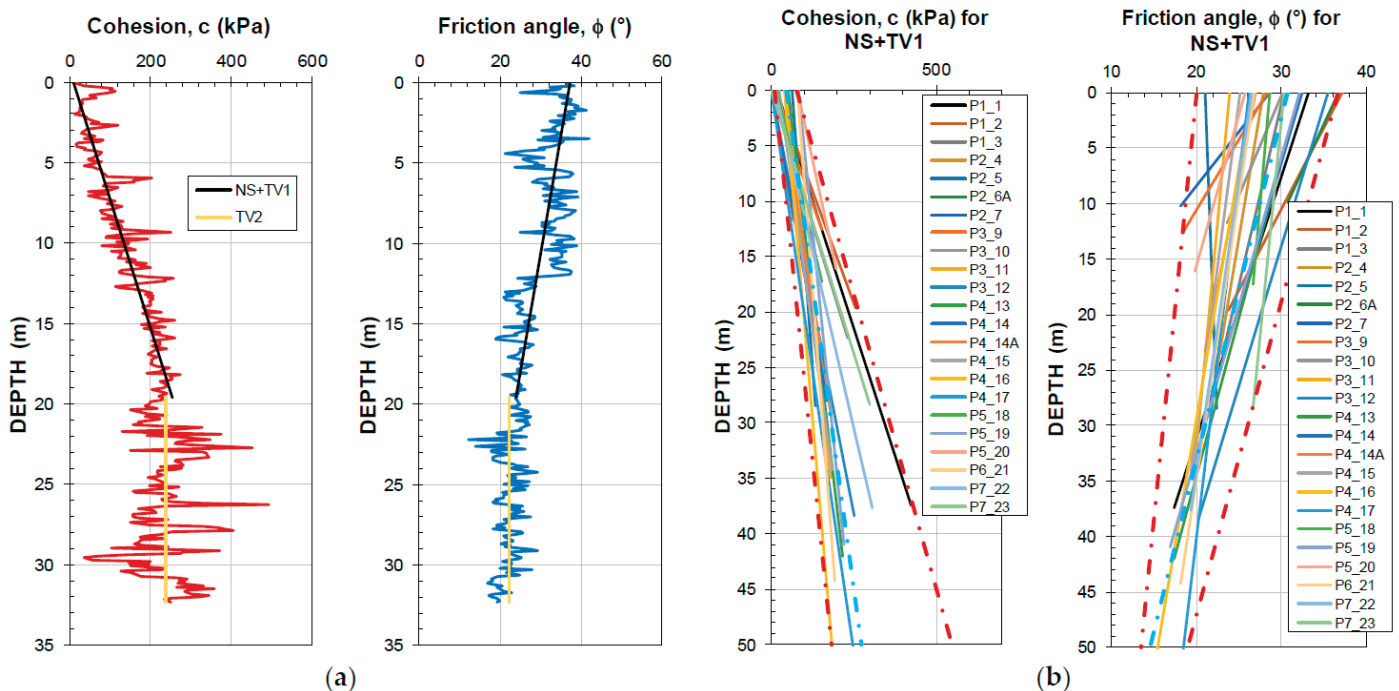


Figure 3: (a) Cohesion and friction angle computed for profile P1_2. (b) Linear variation of cohesion and friction angle for all CPT profiles for NS-TV1 unit; minimum and maximum limits in red and mean value in cyan.

Table 1: Statistical properties of dump units. N, LN & BS dist: Normal, Log-normal and Birnbaum- Saunders distributions; d: depth.

Dump Unit/Property	NS + TV1	TV2	Contact Layer
<i>cohesion: C (kPa)</i>	mean: $4.59 d + 46.15$ min: $3.5 d + 10$ LN dist $\in [0.6, 650]$: $\mu_{LN} = 0.0324 d + 4.034$ $\sigma_{LN} = -0.0063 d + 0.598$	mean: 247.4 min: $4 d - 60$ LN dist $\in [6, 1098]$: $\mu_{LN} = 5.31$ & $\sigma_{LN} = 0.626$	6.0
<i>friction: ϕ (°)</i>	mean: $-0.323 d + 30.69$ min: $-0.13 d + 20$ LN dist $\in [7, 44]$: $\mu_{LN} = -0.241 d + 28.73$ $\sigma_{LN} = -0.071 d + 6.074$	mean: 22.7 min: 16.8 BS dist $\in [8.2, 38.6]$: $\beta = 22.019$ & $\gamma = 0.244$	6.0
<i>Young's modulus: E (MPa)</i>	mean: $3.67 d + 18.51$ min: $2.96 d + 5$ LN dist $\in [1, 354]$: $\mu_{LN} = 0.037 d + 3.577$ $\sigma_{LN} = -0.004 d + 0.377$	mean: 193.9 min: $3.6 d - 52$ LN dist $\in [29.6, 636.5]$: $\mu_{LN} = 3.855 d + 32.046$ $\sigma_{LN} = 0.6195 d + 28.295$	70
<i>saturated density: ρ (t/m³)</i>	mean: $0.0124 d + 1.739$ ($d \leq 12.5$ m) $0.0035 d + 1.869$ ($d > 12.5$ m) min: $0.0035 d + 2.0$ N dist $\in [1.3, 2.2]$: $\mu = 0.0053 d + 1.813$ $\sigma = -0.0008 d + 0.0919$	mean: 2.023 min: $0.0017 d + 2.079$ N dist $\in [1.55, 2.285]$: $\mu = 2.023$ & $\sigma = 0.098$	2.0

of the geological model shown in Figure 1 with a homogeneous structured mesh. To gain accuracy in the areas of interest, this mesh will be replaced by a heterogeneous unstructured mesh. In addition, an assumption was made concerning a contact layer. This layer was not detected in the CPT campaign measurements. This layer may play a major role in the slope stability and in situ observations. Two calculation scenarios have been built from the average values and the minimum bounds of C , ϕ , ρ and E . A third scenario is possible by modelling the properties with the most representative distribution of the measurements. For the TV2 unit (properties not very sensitive to depth), we considered all the data attributed to this unit (2 843 measurements) to find the best distribution. To account for the variation of the statistical parameters with the depth in the NS-TV1 unit, we gathered the measurements in 2 m depth. The measurements are many enough for this unit (population = 6 629) to have enough individuals to analyse. The criterion for selecting the best distribution is a minimization of the square of difference of the theoretical and measured frequencies. As one of the aims of the RAFF project is to develop an operational methodology, we have chosen,

for modelling, only the easily implementable distributions: normal distribution, log-normal and Birnbaum-Saunders. The results of this statistical analysis are reported in Table 1. Note that the log-normal distribution is the one that best represents variations in cohesions and Young's modulus. The same is true for the friction angle except for the TV2 unit, which correlates better with a Birnbaum-Saunders distribution. The normal distribution is the best fit for the spatial distribution of the density masses in the 23 CPT profiles. The probability density function of Birnbaum-Saunders distribution is described by in equation (4).

2.3 | HYDRAULIC DATA

Typically, the position of the water table is poorly known, and it is often determined by a few points where the pore pressure is known, by the hydro-mechanical properties of the studied layers (permeability, porosity, Biot coefficient, dynamic viscosity) and by boundary conditions (null fluid flux, prescribed pore pressure field, inflow, outflow, leakage). Fortunately, the Lake Most site is very well instrumented: the piezometric heights of the

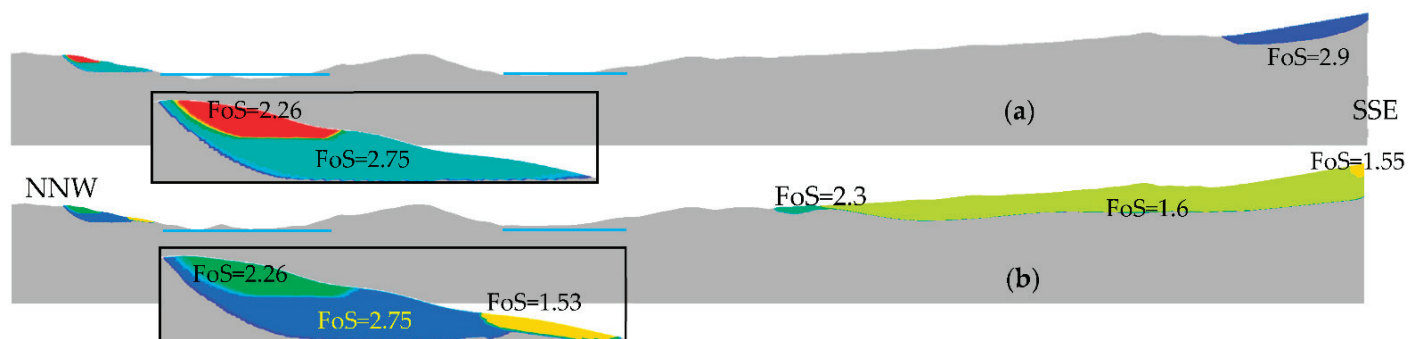


Figure 4: FoS contours for the 4 629 m length NNW-SSE cross section with distributions of C , E , and r in the dump units (a) without contact layer. (b) with contact layer.

water table have been recorded for 93 wells since 2014. The water table DMT has been built from 93 piezometric water levels and lake surface. For slope stability calculations, the position of the water table has been used to calculate the pore pressure and effective stress fields.

3 | RESULTS

Using the strength reduction method produces one global minimum stability state per simulation. This method is applied to the Mohr-Coulomb failure criterion ([8]) by progressively reducing the shear strength of the material to bring the slope to a state of limiting equilibrium. However along a complex slope profile (as the profile of the NNW-SSE cross section: Figure 4), it is interesting to compute multiple minimum states. Instead of excluding different regions of the slope when performing the strength reduction calculation, we have used the ability of Flac3D ([9]) to compute multiple local stability surfaces in a single simulation.

Six calculations were made to estimate the safety factor of the Most site in its current (short-term) situation. The 6 calculations were established from the scenarios of geomechanical properties of dump units (mean values, minimum bounds or statistical distribution) and the presence or not of the contact layer at the bottom of the dump bodies. The contact layer is characterized by very low geomechanical parameters ($c = 6$ kPa and $\phi = 6^\circ$). Some results of these scenarios are shown in Figure 4. First, the 3 scenarios without contact layer produce the same isovalues of FoS because these 6 scenarios differ only in the properties of the dump units and because dump units are not the weakest geologic units. On the other hand, the 3 scenarios including the contact layer change the stability of all dump units (NS-TV1, TV2 and contact layer). This is explained by the very low properties of the contact layer. Areas with FoS of 2.75 (NNW) and 2.9 (SSE) without contact layer have a safety factor of between 1.14 and 1.53 at NNW and between 1.5 and 1.6 at SSE when contact layer is present. The 3D calculations give results compatible with this 2D cross section: FoS = 2.2 and 1.38 without or with contact layer respectively. The global 3D FoS is located on the north bank of Lake Most (Figure 4) at the location where the majority of slope failure stabilization operations took place in the past. In those zones earth and stabilization work were carried out to insure long-term stability.

4 | CONCLUSIONS

In order to establish a reliability method for assessing the long-term stability of flooded open pit mines, a large-scale numerical model of Lake Most was carried out with Flac3D. The model integrated the complex geology of the mine and the dumps, as well as the surface of the water table interpolated from 93 piezometric levels. The results of the 2D and

3D numerical modelling were analyzed as a large scale by calculating global and local safety factors. The results highlighted the reliability of the methodology to combine the geometric, geological and hydraulic models to create a large-scale numerical model, and to identify local and potentially unstable zones. The hypothesis of the presence of a very weak contact layer (at the bottom of the dump bodies) is therefore a strong hypothesis, capable of questioning the stability of the slopes of the site (Most Lake). It should be noted that the contact layer was not detected by the CPT campaign measurements. But neither the absence nor the presence of the contact layer can be confirmed because only 2 profiles are deeper than the dump units in an area to be investigated of more than 8 km². The most realistic hypothesis is probably to consider a partial presence of the contact layer. These new calculations could show whether the reality lies between the minor (with contact layer) and major (without contact layer) scenarios. In this case, the Lake Most site could then be concluded to be stable in the short to medium term.

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SUMMARY

Téměř všechny zbytkové jámy povrchových lomů v Evropě a ve světě jsou koncipovány jako hydrické rekultivace, spočívající v jejich zatopení vodou. Tato umělá jezera jsou v současnosti (a v budoucnu) určena k rekreačním účelům, ale také k alternativnímu energetickému a průmyslovému využití (řeší projekt SUMAD RFCS). Pro zajištění bezpečného využívání těchto lokalit je nutné vyhodnotit potenciální riziko nestability svahů, které je zároveň rizikem dlouhodobým – trvajícím i po vlastním zatopení zbytkové jámy. Touto problematikou se na Evropské úrovni zabývá mezinárodní projekt RFCS RAFF. V rámci tohoto projektu bylo detailně analyzováno jezero Most, které bylo pro svůj rozsah a dostupnost vstupních dat vybráno jako modelová lokalita pro tvorbu velkorozměrového geotechnického 3D modelu. Vytvoření 3D geotechnického modelu v takovém měřítku byl náročný a složitý úkol, který vycházel z digitálního modelu terénu, sestaveného během podrobného LiDARového průzkumu v srpnu 2019. Geologický 3D model lokality byl vytvořen v 50 m gridu, zahrnuje 14 geologických jednotek, včetně antropogenních vrstev a zlomů - rozsáhlý 3D model je tvořen 11 826 069 prvky (viz obrázek č. 1). Geotechnické vstupy modelu vycházejí z výsledků geotechnického průzkumu provedeného těžkou statickou penetrací. Vlastní stabilitní výpočet byl založen na metodě redukce efektivních

parametrů pevnosti zeminy c a ϕ . V rámci této metody je stupeň stability definován jako parametr, kterým je nutno redukovat skutečné hodnoty parametrů c a ϕ vedoucí ke ztrátě stability. Výsledky byly kalibrovány s dlouhodobými výsledky monitoringu lokality.

Hladina podzemní vody byla v modelu interpolována z 93 monitorovaných piezometrických úrovní. Geotechnické údaje týkající se geologických útvarů in-situ vycházejí z rozsáhlé rešerše historických laboratorních a polních zkoušek provedených v lokalitě. Stabilitně jsou rozhodující geotechnické typy zeminy označené jako Q = primárně kvartérní štěrky, JIL1 = plastické měkké jíly a PJIL = písčité jíly. Zejména v oblastech, kde jsou blízko topografického povrchu, fundamentálně ovlivňují výsledky výpočtu stability svahu. Velká pozornost byla věnována výsypkovým geotechnickým typům zemin, které byly detailně popsány díky masivní kampani penetračních měření (CPT). K dispozici je 9 538 měření CPT, včetně pórových tlaků (u_2), plášťového tření (f_s), odporu na hrotu (q_c), korigovaného odporu na hrotu (q_t), objemové hmotnosti (γ), hustoty (ρ), efektivního pórovitého tlaku (u), celkového napětí (σ_v) a Youngova modulu (E). Posledních 6 parametrů bylo klasicky vypočítáno z dat zaznamenaných v kroku 10 cm a parametrů charakterizujících použitý kužel (poměr netto plochy: $a = 0,8$ a průměr penetrometru $B = 0,0357$ m). Protože se předpokládá, že materiály mají Mohr-Coulombov elastoplastický konstituční model, byly z těchto dat dále vypočteny třecí úhel ϕ a soudržnost c (obrázek č. 3).

Redukce parametrů pevnosti zeminy při výpočtu vytváří jeden globální minimální stav stability na simulaci. Tato metoda se aplikuje na Mohr-Coulombovo kritérium porušení ([8]) postupným snižováním smykové pevnosti materiálu tak, aby se svah dostal do stavu mezní rovnováhy.

Avšak podél složitého geotechnického profilu (jako je profil příčného řezu SSZ-JJV: obrázek č. 4) je zajímavé vypočítat více mezních stavů. Při provádění výpočtu redukce pevnosti jsme použili funkci geotechnického softwaru Flac3D ([9]), umožňující modelovat více smykových ploch v jediné simulaci. Vybrané výsledky těchto simulací jsou znázorněny na obrázku č. 4. Obrázek ilustruje klíčový vliv oslabené kontaktní vrstvy na bázi výsypky ($c = 6$ kPa and $\phi = 6^\circ$) na dlouhodobou stabilitu svahu. Tato vrstva nebyla zjištěna u všech penetračních sond. Pokud tato vrstva není ve výpočtu zohledněna, pohybují se zjištěné hodnoty F_s od 2,26 (NNW) do 2,9 (SSE). Pokud však kontaktní vrstvu na bázi výsypkových těles do výpočtu zahrneme, klesají hodnoty F_s z 2,75 na 1,53 (NNW) a z 2,9 na 1,55 (SSE). 3D výpočty poskytují výsledky kompatibilní s tímto 2D řezem: $F_s = 2,2$ a 1,38 bez nebo s oslabenou zónou.

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